Scanning of hadron cross–section at DAΦNE by analysis of the initial–state radiative events

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Abstract

The initial–state radiative events in electron–positron annihilation into hadrons at DAΦNE have been considered. The corresponding cross–section with the full first order radiative corrections has been calculated. The analytical calculations take into account the realistic angular acceptance and energy cut of DAΦNE photon detector.

It is the general viewpoint at present that the dominant error in SM fits arises due to fairly large uncertainity in hadron contribution to vacuum polarization (HVP). The limited knowledge of the HVP affects also the precise determination of the muon anomalous magnetic moment [1,2]. The main problem is connected with the low and intermediate energies because in these regions HVP cannot be computed by means QCD. The only possibility is to reconstruct it by dispersion intrgral using the measured total cross–section of the process $e^+e^- \to hadrons$ at continuously varying energies. Therefore, there exists an eminent physical reason to scan the total hadronic cross–section in the region up to a few GeV.

For this goal one can exploit the high luminosity of oncoming e^+e^- colliders which operate at fixed energy and use the process of radiative return to lower energies due to initial–state photon emission [3–5]:

$$e^-(p_1) + e^+(p_2) - \gamma(k) \to \gamma^*(q) \to H(q)\ , \quad q = p_1 + p_2 - k\ , \tag{1}$$

where H denotes all final hadrons, γ^* is the intermediate heavy photon. We write the final photon on the left side of (1) to emphasize that it radiates from the initial state. The total hadronic cross–section σ_t of the process (1), that defines the imaginary part of the HVP, depends on the heavy photon virtuality

$$q^2 = s(1-x)\ , \quad s = 4\varepsilon^2\ , \quad x = \frac{\omega}{\varepsilon}\ ,$$

where $\omega(\varepsilon)$ is the photon (electron) energy. Therefore, by measuring the photon energy fraction x one can extract the distribution $\sigma(s(1-x))$.

In the case of final–state radiation process

$$e^-(p_1) + e^+(p_2) \to \gamma^*(p_1+p_2) \to \gamma(k) + H(q) \tag{2}$$

the quantity σ_t depends on s, while the hadron invariant mass q^2 leaves the same. Cosequently, in order to scan the total hadronic cross–section it needs to use only events with initial–state

radiation. The events with final–state radiation have to be considered as a background. In Ref.[5] authors suggested to restrict the angular hadron phase space to get rid of the final–state radiation background. On our opinion the such procedure is not adequate to main goal, because the exctracted in this case experimental cross–section must depends on the restriction parameters, whereas the quantity σ_t (which enters into dispersion integral) depends, by definition, on the heavy photon invariant mass only.

In present work we calculate the quantity σ_t for the process (1) and radiative correction to it, using the realistic conditions of the tagged photon detector (PD) at DAΦNE collider. The angular acceptance of the DAΦNE PD covers all phase space except for a two symmetrical cones with the opening angle $2\theta_0$ along both, the electron and the positron beam directions. Such kind of angular acceptance is just opposite to one used in [4] where the PD has covered narrow cone along the electron (or the positron) beam direction. Besides, the realistic DAΦNE PD selects the events with only one hard photon hitting it, and the coresponding energy cutoff parameter is Δ. Under the DAΦNE conditions

$$\theta_0 = 10^\circ \ , \quad \Delta = 4\cdot 10^{-2} \ , \tag{3}$$

and the radiative corrected total hadronic cross–section will depend on both, the angular and the energy cutoff parameters.

The Born cross–section of the process (1) can be written as follows [4,6]

$$d\sigma^B = \int\limits_{\Omega(\theta_0)} \sigma(q^2)\frac{\alpha}{2\pi^2}\frac{(s+t_1)^2+(s+t_2)^2}{t_1 t_2}\frac{d^3k}{s\omega} \ , \tag{4}$$

where $\Omega(\theta_0)$ covers the angular acceptance of PD, and $t_{1,2} = -2(kp_{1,2})$. The normalizaion cross–section under integral sign on the right side of Eq.(4) can be expressed in terms of the ratio R of the total hadronic and muonic cross–sections:

$$\sigma(q^2) = \frac{4\pi\alpha^2}{3q^2|1-\Pi(q^2)|^2}R(q^2)(1+\frac{2\mu^2}{q^2})\sqrt{1-\frac{4\mu^2}{q^2}} \ , \quad R(q^2) = \frac{\sigma_t(e^+e^-\rightarrow hadrons)}{\sigma_t(e^+e^-\rightarrow \mu^+\mu^-)} \ , \tag{5}$$

where μ is the muon mass. The lepton contribution into vacuum polarization $\Pi(q^2)$ is the known function [1] and will not be specific here. The angular integration of Eq.(4) gives

$$d\sigma^B = \frac{\alpha}{2\pi}\sigma(q^2)2[\frac{1+(1-x)^2}{x}\ln\frac{1+\cos\theta_0}{1-\cos\theta_0} - x\cos\theta_0]dx \ . \tag{6}$$

In the Born approximation the cross–section of the process (1) depends on the angular cutoff parameter only. Looking at Eq.(6) we see that the measurement of that cross–section at different values of the tagged photon energy fraction x allows to extract the quantity $\sigma(e^+e^-\rightarrow hadrons)$ at different effective collision energies $s(1-x)$.

The high precision measurement of the total hadronic cross–section requires to adequate theoretical calclations. These last have to take into account at least the first order radiative correction (RC). The RC to $d\sigma^B$ includes the contributions due to aditional virtual and real soft (with the energy less then $\varepsilon\Delta$ $,\omega < \varepsilon\Delta$) photon emission in all angular phase space as well as due to hard ($\omega > \varepsilon\Delta$) photon emission in the region where the PD does not record it.

When calculating the RC to the Born cross–section we suggest that $\sigma(q^2)$ is the flat enough function of its argument, such that the conditions

$$\frac{\Delta}{\sigma(q^2)}\frac{d\sigma}{d\ln(q^2)} \ll 1 \ , \quad \frac{\theta_0^2}{\sigma(q^2)}\frac{d\sigma}{d\ln(q^2)} \ll 1 \tag{7}$$

are satisfied. These conditions permit to apply the soft photon approximatiom and the quasireal electron method [8] for a description of only wide resonance contributions into cross–section.

The virtual and soft photon corretions can be computed using the results of work [7] where one–loop corrected Compton tensor with a heavy photon was calculated for the scattering channel. In order to reconstruct the corresponding results for the annihilation channel it is enough to change (in accordance with the notation used here)

$$p_2 \to -p_2 \ , \quad u \to s \ , \quad t \to t_1 \tag{8}$$

in all formulae of the Ref.[7].

Thus, the contribution of virtual and soft photon emission into the RC to Born cross–section can be written as follows

$$d\sigma^{V+S} = \int\limits_{\Omega(\theta_0)} \sigma(q^2)\frac{\alpha^2}{4\pi^3}[\rho\frac{(s+t_1)^2+(s+t_2)^2}{t_1t_2} + T]\frac{d^3k}{s\omega} \ , \tag{9}$$

$$\rho = 4(l_s-1)\ln\Delta + 3(l_s+\ln(1-x)) - \frac{\pi^2}{3} - \frac{9}{2} \ , \quad l_s = \ln\frac{s}{m^2} \ , \tag{10}$$

$$T = \frac{3}{2}T_g - \frac{1}{8q^2}\{T_{11}(s+t_1)^2 + T_{22}(s+t_2)^2 + (T_{12}+T_{21})[s(s+t_1+t_2) - t_1t_2]\} \ , \tag{11}$$

where m is the electron mass. For quantities T_g and T_{ik} see [7], bearing in mind substitution (8). It needs to note only that under the DAΦNE conditions $|t_{1,2}|_{min} \approx \varepsilon^2\theta_0^2 \gg m^2$, therefore one have to omitt terms proportional to m^2 in both T_g and T_{ik}.

The Born–like structure, that contains multiplier ρ, on the right side of Eq.(9) absorbs all infrared singularities via quantity $\ln\Delta$. In the limiting case

$$|t_1| = 2\varepsilon^2(1-c) \approx \varepsilon^2\theta_0^2 \ll s, \ |t_2| \ ; \quad t_2 = -sx \ , \quad q^2 = s(1-x) \ ,$$

which corresponds to events with the tagged photon detected very close to the cutoff angle θ_0 along the electron beam direction, the expression into parenthesis on the right side of Eq.(9) reads

$$2\rho\frac{1+(1-x)^2}{x^2(1-c)} + \frac{2}{x(1-c)}\{\frac{1+(1-x)^2}{x}[\ln(1-x)\ln\frac{x^2(1-c)}{2(1-x)} - 2f(x)] + \frac{2-x^2}{2x}\} \ , \tag{12}$$

$$f(x) = \int\limits_0^x \frac{dz}{z}\ln(1-z) \ , \quad c = \cos\theta \ ,$$

where θ is the angle between vectors $\vec{k}$ and $\vec{p_1}$. For events with recorded photon very close to the cutoff angle θ_0 along the positron beam direction it needs to change c in (12) by $-c$.

To compute the RC due to invisible hard photon radiated along the electron beam direction inside the cone with the opening angle $2\theta_0$ we can use the quasireal electron method [8]. In accordance with this method the corresponding contribution into cross–section has a form

$$d\sigma_1^H = \int d\sigma^B(p_1-k_1, k, p_2)dW_{p_1}(k_1) \tag{13}$$

where $d\sigma_1^H$ is the cross–section of the process

$$e^-(p_1) + e^+(p_2) - \gamma(k) - \gamma(k_1) \to \gamma^* \to H(q) \tag{14}$$

provided the additional hard photon $\gamma(k_1)$ is emitted along the electron beam direction.

The expression for the radiation probability $dW(k_1)$ is well known [8], and under the DAΦNE conditions it may be written as folows

$$dW(k_1) = \frac{\alpha}{2\pi}P(1-z, \ln\frac{\varepsilon^2\theta_0^2}{m^2})dz \ , \quad P(1-z,L) = \frac{1+(1-z)^2}{z}L - \frac{2(1-z)}{z} \ , \tag{15}$$

where z is the energy fraction of invisible photon, and we use approximation: $2(1-\cos\theta_0) = \theta_0^2$ in argument of logarithm.

The shifted Born cross–section on the right side of Eq.(13) is defined by the formula

$$d\sigma^B(p_1-k_1,k,p_2) = \int\limits_{\Omega(\theta_0)} \frac{\alpha}{2\pi}\sigma(q_1^2)\frac{(1-z)^2(s+t_1)^2 + ((1-z)s+t_2)^2}{(1-z)^2t_1t_2}\frac{d^3k}{s\omega} \ , \tag{16}$$

$$q_1 = (1-z)p_1 + p_2 - k \ .$$

After integration over the invisible photon energy fraction z on the right side of Eq.(13) we derive

$$d\sigma_1^H = \frac{\alpha}{2\pi}\int\limits_{\Delta}^{z_m} dzP(1-z, \ln\frac{\varepsilon^2\theta_0^2}{m^2})d\sigma^B(p_1-k_1,k,p_2) \ , \tag{17}$$

where the upper limit of integration is defined by condition $q_1^2 \geq 4m_\pi^2$ (m_π is the pion mass) and reads

$$z_m = \frac{2(1-x-\delta)}{2-x(1-c)} \ , \quad \delta = \frac{4m_\pi^2}{s} \ .$$

The corresponding expression for $d\sigma_2^H$, when the additional invisible hard photon is emitted along the positron beam direction, can be obtained from Eq.(17) by substitution $p_1 \leftrightarrow p_2$ in $d\sigma^B$ and $c \to -c$ in z_m. Because the angular acceptance of the DAΦNE PD is symmetrical respect to change $c \to -c$, $d\sigma_1^H = d\sigma_2^H$, and the full RC to the Born cross–section reads

$$d\sigma^{RC} = d\sigma^{V+S} + 2d\sigma_1^H \ . \tag{18}$$

It is useful to rewrite the function $P(1-z,L)$ where $L = l_s + \ln\frac{\theta_0^2}{4}$, that enters into $d\sigma_1^H$ in the following form

$$P(1-z,L) = P_1(1-z,L) - 2G - \delta(z)[(\frac{3}{2} + 2\ln\Delta)L - 2\ln\Delta] \ , \tag{19}$$

$$P_1(1-z,L) = [\frac{1+(1-z)^2}{z}\theta(z-\Delta) + \delta(z)(\frac{3}{2}+2\ln\Delta)]L \ , \ G = \frac{1-z}{z}\theta(z-\Delta) + \delta(z)\ln\Delta \ ,$$

where the quantity $(\alpha/2\pi)P_1(y,L)$ is the well known first order electron structure function [9], and simultaniously suppose the lower limit of z–integration in $d\sigma_1^H$ to be equal to zero. Then the measured cross–section of the process (1) under the DAΦNE conditions can be written as follows

$$d\sigma = d\sigma^B(p_1,k,p_2)\{1 + \frac{\alpha}{2\pi}[(3+4\ln\Delta)\ln\frac{4}{\theta_0^2} + 3\ln(1-x) - \frac{\pi^2}{3} - \frac{9}{2}]\} \tag{20}$$

$$+\frac{\alpha}{2\pi}\{\frac{\alpha}{2\pi^2}\int\limits_{\Omega(\theta_0)}\sigma(q^2)T\frac{d^3k}{s\omega} + 2\int\limits_0^{z_m}dz[P_1(1-z,L) - G]d\sigma^B((1-z)p_1,k,p_2)\} \ .$$

The term containing the product of logarithms of the energy and the angle cutoff parameter arises because we do not permit for the additional hard photon to appear inside PD. We can resum the main contributions on the right side of Eq.(20) in all orders of perturbation theory and write the master formula in the form

$$d\sigma = \int dz_1 \int dz_2 d\sigma^B(z_1, z_2)\{D(z_1, L)D(z_2, L) - \frac{\alpha}{2\pi}[\delta(1-z_1)G(1-z_2) + \delta(1-z_2)G(1-z_1)]\}\Theta_{12}$$

$$+ d\sigma^B(1,1)[\frac{\alpha}{2\pi}(3\ln(1-x) - \frac{\pi^2}{3} - \frac{9}{2}) + \exp{(\beta l_s)}(1 - \exp{(\beta \ln\frac{\theta_0^2}{4})})] \tag{21}$$

$$+\frac{\alpha^2}{4\pi^2}\int\limits_{\Omega(\theta_0)} \sigma(q^2)T\frac{d^3k}{s\omega} \ , \quad d\sigma^B(z_1, z_2) = d\sigma^B(z_1p_1, k, z_2p_2) \ , \quad \beta = \frac{2\alpha}{\pi}(\frac{3}{2} + \ln\Delta) \ ,$$

where D(z,L) is the full electron sructure function. Teta–function Θ_{12} under integral sign defines the integration limits over variables z_1 and z_2 provided

$$(z_1p_1 + z_2p_2 + k)^2 \geq 4m_\pi^2 \ .$$

The corresponding limits of integration can be written as follows

$$1 > z_2 > \frac{2\delta + z_1x(1-c)}{2z_1 - x(1+c)} \ , \quad 1 > z_1 > \frac{2\delta + x(1+c)}{2 - x(1-c)} \ . \tag{22}$$

The cross–section (21) corresponds to such event selection when one hard photon with the energy fraction x hitting PD and accompaning with the arbitrary number of soft photons with the energy fraction up to Δ for every ones inside PD is included as an event. If we want select events when the energy fraction of all soft photons inside PD does not exeed Δ we have to change β by $\bar{\beta} = \frac{\alpha}{2\pi}(\frac{3}{4} - C + \ln\Delta)$, where C is the Euler costant and write

$$\frac{1}{\Gamma(1+\frac{\alpha}{2\pi}l_s)} - \frac{\exp(\bar{\beta}\ln\frac{\theta_0^2}{4})}{\Gamma(1+\frac{\alpha}{2\pi}L)}$$

instead of $1 - \exp(\beta\ln\frac{\theta_0^2}{4})$ on the second line of Eq.(21).

The master formula (21) takes into account only photonic RC. It can be genegalized in such a way to include also the leading corrections due to electron–positron pair production. We will assume that inside PD only soft pairs (with the energy fraction less than Δ) can be present, while outside PD both, the soft and hard ones. In this case the corresponding generalization can be carried out by insertion of effective electromagnetic coupling [9] instead of αl_s and αL

$$\alpha l_s \rightarrow -3\pi\ln(1 - \frac{\alpha}{3\pi}l_s) \ , \quad \alpha L \rightarrow -3\pi\ln(1 - \frac{\alpha}{3\pi}L) \ , \tag{23}$$

in the electron structure functions and exponents on the right side of Eq.(21). Besides this we have to represent the electron structure function as a sum of nonsinglet and singlet parts [9]. Note that the nonsinglet part can be written in iterative as well as in exponentiated form, whereas the singlet one has only iterative form. For the electron structure functions see [9,10].

To extract the radiative corrected hadronic cross–section from the corresponding experimental data of DAΦNE collider with a few tenth percent accuracy it is enough to use Eq.(20). In this case we can expand the quantity $\sigma(q^2)$ as

$$\sigma(q^2) = \sigma_0(q^2) + \frac{\alpha}{2\pi}\sigma_1(q^2) \tag{24}$$

and obtain σ_0 and σ_1 by application of the simple iterative procedure to Eq.(20) bearing in mind that the cross–section on the left side of this equation is measured by experiment. The corresponding equation in zero approximation

$$d\sigma^{exp} = \frac{\alpha}{2\pi^2} \int\limits_{\Omega(\theta_0)} \sigma_0(q^2) \frac{(s+t_1)^2 + (s+t_2)^2}{t_1 t_2} \frac{d^3k}{s\omega} \tag{25}$$

allows to extract the dependence $\sigma_0(q^2)$ in wide interval of variable q^2. In the first approximation the equation for σ_1 reads

$$\int\limits_{\Omega(\theta_0)} \Big\{ \Big[\sigma_1(q^2) + \sigma_0(q^2)[(3 + 4\ln\Delta)\frac{4}{\theta_0^2} + 3\ln(1-x) - \frac{\pi^2}{3} - \frac{9}{2}] \Big] \frac{(s+t_1)^2 + (s+t_2)^2}{t_1 t_2}$$

$$+ \sigma_0(q^2)T + 2\int\limits_0^{z_m} \sigma_0(q_1^2)[P_1(1-z, L) - G] \frac{(1-z)^2(s+t_1)^2 + ((1-z)s + t_2)^2}{(1-z)^2 t_1 t_2} dz \Big\} = 0 \ . \tag{26}$$

This equation can be solved numerically respect to function $\sigma_1(q^2)$.

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